Dominance of a Dynamical Measure and Disappearance of the Cosmological Constant

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Abstract

We consider an action which consists of two terms: the first $S_1 = \int L_1 \Phi d^4x$ and the second $S_2 = \int L_2 \sqrt{-g} d^4x$ where Φ is a measure which has to be determined dynamically. S_1 satisfies the requirement that the transformation $L_1 \to L_1 + const.$ does not effect equations of motion. In the first order formalism, a constraint appears which allows to solve $\chi = \Phi/\sqrt{-g}$. Then, in a true vacuum state (TVS), $\chi \to \infty$ and in the conformal Einstein frame no singularities are present, the energy density of TVS is zero without fine tuning of any scalar potential in S_1 or S_2. When considering only a linear potential for a scalar field ϕ in S_1, the continuous symmetry $\phi \to \phi + const$ is respected. Surprisingly, in this case SSB takes place while no massless ("Goldstone") boson appears.

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1. Introduction.

One of the greatest puzzles of modern physics is the Cosmological Constant Problem (CCP) which consists of the fact that present day vacuum energy is zero or so small in Planck units.

In a series of papers [1]-[4] we have developed the so called Nongravitating Vacuum Energy (NGVE) theory, where the action of the theory is assumed to be

$$S = \int \Phi L d^4 x \tag{1}$$

Φ being a total derivative of something and also a scalar density, as for example can be achieved by constructing it out of four scalar fields $\varphi_a \quad (a = 1, ..., 4)$:

$$\Phi \equiv \varepsilon_{abcd} \varepsilon^{\mu\nu\alpha\beta} (\partial_\mu \varphi_a)(\partial_\nu \varphi_b)(\partial_\alpha \varphi_c)(\partial_\beta \varphi_d) \tag{2}$$

In this case L can be changed as in $L \rightarrow L + const$ without affecting the equations of motion. Notice that Φ is a measure independent of $g_{\mu\nu}$ as opposed to the case of GR where the measure of integration is $\sqrt{-g}$. In what follows we will call Φ the *Dynamical Measure* (DM) since its value is dynamically determined in terms of all the fields of the theory through the equations of motion as we will see.

In Refs. [2]-[4] we have considered

$$L = -\frac{1}{\kappa} R(\Gamma, g) + L_m \tag{3}$$

where the matter Lagrangian density L_m does not contain φ_a dependence and $R(\Gamma, g)$ is the scalar curvature $R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ of the space-time of the affine connection $\Gamma^\mu_{\alpha\beta}$: $\quad R_{\mu\nu}(\Gamma) = R^\alpha_{\mu\nu\alpha}(\Gamma)$, $\quad R^\lambda_{\mu\nu\sigma}(\Gamma) \equiv \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma}\Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu}\Gamma^\alpha_{\mu\sigma}$. If in addition

to this we assume that L_m contains 4-index field strength and also scalar fields with generic potentials with no special fine tuning imposed on them, we obtain generically [4] that Eqs. (1)-(3) solve the cosmological constant problem.

Here we will see that it is possible to extend considerably the class of theories where the CCP is solved in the same spirit. In fact we will see that it is possible to add an *explicit* cosmological constant term to Eq.(1), or even more general, a coupling of a scalar field ϕ to $\sqrt{-g}$ of the form

$$\int U(\phi)\sqrt{-g}d^4x \tag{4}$$

without destroying the mechanism that drives the effective cosmological constant to zero which can work even in the absence of a 4-index field strength. Further generalizations and extensions of the theory can also be made, without destroying the basic mechanism which solves the CCP.

2. Dynamical measure dominance in the true vacuum state and solution of the CCP

To demonstrate how the theory works, we start here from the simplest model in the first order formalism including scalar field ϕ and gravity according to the prescription of the NGVE principle and in addition to this we include the standard cosmological constant term. Possible reasons for such kind of structure will be discussed at the end of this letter. So, we consider an action

$$S = \int L_1 \Phi d^4x + \int \Lambda\sqrt{-g}d^4x \tag{5}$$

where

$$L_1 = -\frac{1}{\kappa}R(\Gamma, g) + \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi) \tag{6}$$

Performing the variation with respect to the measure fields φ_a (see Eq.(2)) we obtain equations $A^{\mu}_{a}\partial_\mu L_1 = 0$ where $A^{\mu}_{b} = \varepsilon_{acdb}\varepsilon^{\alpha\beta\gamma\mu}(\partial_\alpha\varphi_a)(\partial_\beta\varphi_c)(\partial_\gamma\varphi_d)$. It is easy to check (see [1]-[3]) that if $\Phi \neq 0$, then it follows from the last equations that $L_1 = M = const$

Varying the action (5) with respect to $g^{\mu\nu}$ we get

$$\Phi(-\frac{1}{\kappa}R_{\mu\nu}(\Gamma) + \frac{1}{2}\phi_{,\mu}\phi_{,\nu}) - \frac{1}{2}\sqrt{-g}\Lambda g_{\mu\nu} = 0 \tag{7}$$

Contracting Eq.(7) with $g^{\mu\nu}$ and using equation $L_1 = M$ we obtain the constraint

$$M + V(\phi) - \frac{2\Lambda}{\chi} = 0 \tag{8}$$

where we have defined the scalar field $\chi \equiv \Phi/\sqrt{-g}$.

The scalar field ϕ equation is

$$(-g)^{-1/2}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + \sigma_{,\alpha}\phi^{,\alpha} + V' = 0 \tag{9}$$

where $\sigma \equiv \ln\chi$ and $V' \equiv dV/d\phi$.

The solution of the equation obtained by variation of the connection $\Gamma^{\alpha}_{\mu\nu}$ may be represented in the form $\Gamma^{\alpha}_{\mu\nu} = \{^{\alpha}_{\mu\nu}\} + \Sigma^{\alpha}_{\mu\nu}(\sigma)$ where $\{^{\alpha}_{\mu\nu}\}$ are the Christoffel's connection coefficients and $\Sigma^{\alpha}_{\mu\nu}$ is a function of derivatives of σ. After making use the λ-symmetry [5] of the action (5)

$$\Gamma'^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} + \delta^{\alpha}_{\mu}\lambda_{,\nu} \tag{10}$$

the antisymmetric part of $\Sigma^{\alpha}_{\mu\nu}(\sigma)$ can be eliminated and we get

$$\Sigma^{\alpha}_{\mu\nu}(\sigma) = \frac{1}{2}(\delta^{\alpha}_{\mu}\sigma_{,\nu} + \delta^{\alpha}_{\nu}\sigma_{,\mu} - \sigma_{,\beta}g^{\alpha\beta}g_{\mu\nu}) \tag{11}$$

The derivatives of the field σ enter both the gravitational equation (7) (through the connection) and in the scalar field equation (9). By a conformal transformation

$$g_{\mu\nu} \rightarrow \overline{g}_{\mu\nu} = \chi g_{\mu\nu}; \qquad \phi \rightarrow \phi \tag{12}$$

to an "Einstein picture" and using the constraint (8) we obtain the canonical form of equations for the scalar field ϕ

$$(-\overline{g})^{-1/2}\partial_{\mu}(\sqrt{-\overline{g}} \quad \overline{g}^{\mu\nu}\partial_{\nu}\phi) + V'_{eff}(\phi) = 0 \tag{13}$$

and the gravitational equations in the Riemannian space-time with metric $\overline{g}_{\mu\nu}$

$$R_{\mu\nu}(\overline{g}_{\alpha\beta}) - \frac{1}{2}\overline{g}_{\mu\nu}R(\overline{g}_{\alpha\beta}) = \frac{\kappa}{2}T^{eff}_{\mu\nu}(\phi) \tag{14}$$

where $T^{eff}_{\mu\nu}(\phi) = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\overline{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\overline{g}^{\alpha\beta} + V_{eff}(\phi)\overline{g}_{\mu\nu}$,

$$V_{eff}(\phi) = \frac{1}{4\Lambda}[M + V(\phi)]^2 \tag{15}$$

and

$$V'_{eff}(\phi) = \frac{1}{2\Lambda}[M + V(\phi)]V'(\phi) \tag{16}$$

We see that for any analytic function $V(\phi)$, the effective potential in the Einstein picture has an extremum, i.e. $V'_{eff} = 0$, either when $V' = 0$ or $V + M = 0$. The extremum $\phi = \phi_1$ where $V'(\phi_1) = 0$ has nonzero energy density $[M + V(\phi_1)]^2/4\Lambda$ if a fine tuning is not assumed. In contrast to this, if $\Lambda > 0$, the state $\phi = \phi_0$ where $V(\phi_0) + M = 0$ is the absolute minimum and therefore ϕ_0 is a true vacuum with

zero cosmological constant without any fine tuning. A mass square of the scalar field describing small fluctuations around ϕ_0 is

$$m^2 = \frac{1}{2\Lambda}[V'(\phi_0)]^2 \tag{17}$$

Let us now consider a couple of models for $V(\phi)$ and some interesting effects associated with them.

3. Model with continuous symmetry related to the NGVE principle and SSB without generating a massless scalar field

It is interesting to see what happens in the model for the choice $V = J\phi$, where J is some constant. Then the action (5), (6) is invariant (up to the integral of total divergence) under the shift $\phi \to \phi + const$ which is in fact the symmetry $V \to V + const$ related to the NGVE principle. Notice that if we consider the model with complex scalar field ψ, where ϕ is the phase of ψ, then the symmetry $\phi \to \phi + const$ would be the $U(1)$ - symmetry.

The effective potential (15) in such a model has the form

$$V_{eff} = \frac{1}{2}m^2(\phi - \phi_0)^2 \tag{18}$$

where $\phi_0 = -M/J$ and $m^2 = J^2/2\Lambda$. We see that the symmetry $\phi \to \phi + const$ is spontaneously broken and mass generation is obtained. However, *no massless scalar field results from the process of SSB in this case, i.e. Goldstone theorem does not apply here.*

This seems to be a special feature of the NGVE - theory which allows: 1) To start with linear potential $J\phi$ without destroying the shift symmetry $\phi \to \phi + const$, present in the $\partial_\mu\phi\partial^\mu\phi$ piece, due to the coupling to the dynamical measure (2). This shift symmetry is now a symmetry of the action up to a total divergence. 2) This potential gives rise to an effective potential $(M + J\phi)^2/4\Lambda$. The constant of integration M being responsible for the SSB.

Similar effect can be obtained even in the pure NGVE - theory as in Eq.(1) (without introduction of an explicit Λ-term) but with the use of 4-index field strength condensate. The possibility of constructing spontaneously broken $U(1)$ models which do not lead to associated Goldstone bosons is of course of significant physical relevance. One may recall for example the famous $U(1)$ problem in QCD [6]. Also the possibility of mass generation for axions is of considerable interest. These issues will be developed further in elsewhere [7].

4. Selfinteraction and SSB from quadratic scalar field potential

If we choose the quadratic potential $V(\phi) = \frac{\mu^2}{2}\phi^2$, which in the usual theory is associated with free field theory in curved space-time, we obtain here a very different result, i.e. that the effective potential in the Einstein picture is

$$V_{eff} = \frac{\mu^4}{16\Lambda}(\phi^2 - \phi_0^2)^2 \tag{19}$$

where $\phi_0^2 = -2M/\mu^2$

We see that if $\Lambda > 0$, spontaneous breaking of the discrete symmetry $\phi \to -\phi$ takes place if $M\mu^2 < 0$. Yet, the vacuum energy at the absolute minimum $\phi = \pm|\phi_0|$

is identically zero. Furthermore, the mass of the scalar field is $m^2 = \frac{\mu^4}{2\Lambda}\phi_0^2 = -\frac{M\mu^2}{\Lambda}$ which as we see depends on the integration constant M. The mass m is therefore a "floating" physical parameter, since M does not appear in the original Lagrangian but it is determined by initial conditions of the Universe.

If ϕ is replaced by a complex field ψ and ϕ^2 by $\psi^*\psi$ we obtain the SSB of a continuous symmetry with standard consequences (as opposed to example of Sec.3). In addition, a model of cosmology that can include an inflationary phase taking place in a false vacuum and transition to a zero cosmological constant phase is obtained without fine tuning.

5. Discussion

Further generalizations, like considering a term of the form $\int U(\phi)\sqrt{-g}d^4x$ instead of $\Lambda \int \sqrt{-g}d^4x$, the possibility of coupling of scalar fields to curvature, etc. do not modify the qualitative nature of the effects described here and they will be studied in a more detailed publication [7]. For example, even in the presence of $V_1(\phi)$ the resulting effective potential vanishes when $V(\phi_0)+M=0$ and it goes as $V_{eff} = \frac{1}{4U(\phi_0)}(V+M)^2$ in the region $V+M \sim 0$.

Some sources of the type of structure being considered here are suggested. First of all, as mentioned in Ref.[1], if we start with only $S_1 = \int L_1 \Phi d^4x$ as a fundamental theory, possible radiative corrections in the effective action are severely constrained due to the existence of the infinite dimensional symmetry [1] (up to a total divergence) which consists of the infinitesimal shift of the fields φ_a by an arbitrary infinitesimal

function of the total Lagrangian density L_1

$$\varphi_a \rightarrow \varphi_a + \epsilon g_a(L_1), \qquad \epsilon \ll 1o \tag{20}$$

This symmetry prevents the appearance of terms of the form $f(\chi)\Phi$ in the effective action with the single possible exception of $f(\chi) = c/\chi$ where a scalar c is χ independent. This is because in this last case the term $f(\chi)\Phi = c\sqrt{-g}$ is φ_a independent. This possibility gives rise to the cosmological constant term in the effective action as in Eq.(5) while the symmetry (20) is maintained. This can be generalized to possible contributions of the form $\int L_2\sqrt{-g}d^4x$ where L_2 is φ_a independent function of matter fields and gravity if radiative corrections generate a term $f(\chi)\Phi$ with $f(\chi) = L_2/\chi$.

One may question also the possible origin of the measure of integration Φ. It appears to us that an interesting possibility is that this may correspond to a space-filling brane [8], as discussed in Ref.[3]. It may be noticed that if we take $L_1 = const$ in the action $\int L_1\Phi d^Dx$ of the fundamental theory in an arbitrary dimension D, we obtain a purely topological theory which has been interpreted as the nontrivial action for a $D-2$ brane by Zaikov [9]. In our case, the introduction of $L_1 \neq const$ changes drastically the nature of the theory, making it dynamical rather than topological, but may be the geometrical interpretation (or parts of it) could be retained.

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